2-modular representations of finite simple groups as binary codes

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Let F be a finite field of q elements, and G be a transitive group on a finite set Ω. Then there is a G-action on Ω, namely a map $G \times \Omega \longrightarrow \Omega$, $(g, w) \mapsto w^g = gw$, satisfying $w^{gg'} = (gg')w = g(g'w)$ for all $g, g' \in G$ and all $w \in \Omega$, and that $w^1 = 1w = w$ for all $w \in \Omega$. Let $F\Omega = \{f \mid f \colon \Omega \longrightarrow F\}$, be the vector space over F with basis Ω. Extending the G-action on Ω linearly, $F\Omega$ becomes an FG-module called an FG-permutation module. We are interested in finding all G-invariant FG-submodules, i.e., codes in $F\Omega$. The elements $f \in F$ are written in the form $f = \sum_{w\in\Omega} a_w \chi_w$ where χ_w is a characteristic function. The natural action of an element $g \in G$ is given by $g(\sum_{w\in\Omega} a_w \chi_w) = \sum_{w\in\Omega} a_w \chi_{g(w)}$. This action of G preserves the natural bilinear form defined by

$$\langle \sum_{w\in\Omega} a_w \chi_w, \sum_{w\in\Omega} b_w \chi_w \rangle = \sum_{w\in\Omega} a_w b_w.$$

In particular, and by way of illustration we determine all linear codes of length 100 over $\mathbb{F}_2$ which admit the simple group HS of Higman-Sims. By group representation theory it is proved that they can all be understood as submodules of the permutation module $F\Omega$ where Ω denotes the vertex set of the rank-3 graph associated with the simple group HS of Higman-Sims.