Some open conjectures on linear codes from finite planes

J.D. Key

Abstract

We consider some open questions regarding codes arising from the row span over a finite field $\mathbb{F}_p$ of a lines by points incidence matrix of a finite projective or affine plane, and in particular those coding-theoretic properties that may distinguish a non-desarguesian from a desarguesian plane, including bounds on the dimension of the p-ary code C, the minimum weight and nature of the minimum weight vectors of the hull, $C \cap C^\perp$, and the nature of the minimum weight vectors for an affine plane.

In particular, we show that projective Hall planes of even order q provide an infinite class of planes that confirm the Hamada-Sachar conjecture that the code from the desarguesian plane has the smallest dimension and that these planes, and their duals, are not tame, i.e. there are words in the hull of weight $2q$ that are not the difference of the incidence vectors of two lines [2].

We also exhibit words of weight 16 in several non-desarguesian affine planes of order 16 that are not incidence vectors of lines. This result is from a computer study of the binary codes and their hulls of all the known non-desarguesian projective planes of order 16 [1].

The new results were all obtained by an examination of possible words in the hulls of the projective planes.

References

[1] Dina Ghinelli, Marialuisa J. de Resmini, and Jennifer D. Key, *Minimum words of codes from affine planes*, J. Geom. **91** (2008), 43–51.

[2] J. D. Key, T. P. McDonough, and V. C. Mavron, *Codes from Hall planes of even order*, J. Geom. **105** (2014), 33–41.