Axial algebras and their Miyamoto groups

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An axial algebra is a commutative (but not necessarily associative) algebra generated by semisimple idempotents (its axes), for each of which the eigenspaces multiply according to a restrictive fusion rule. An example is furnished by the idempotents of a Jordan algebra and their associated Pierce decompositions; but the motivating example is the Griess algebra for the Monster, as recast by Conway and embedded in the Monster Vertex Operator Algebra of Frenkel, Lepowsky, and Meurman. As with a Jordan algebra, the fusion rule can often be refined to a $\mathbb{Z}_2$-grading; and the associated automorphisms of order 2, the Miyamoto involutions, say a great deal about the structure of the algebra. Indeed, for certain special rules they lead to a full classification. For the Griess algebra, the generated Miyamoto group is the Monster in its natural action on the VOA. The techniques and results can be applied to other VOA and related algebras, as well. (The results discussed will include work of Felix Rehren, Sergey Shpectorov, and Tom De Medts.)